Applying Composite Disturbances to Control Breakup of Capillary Circular Liquid Jets

Mina Rohani*, Derek Dunn-Rankin, and Faryar Jabbari
Department of Mechanical and Aerospace Engineering
University of California Irvine
Irvine, CA 92697 USA

Abstract

In this paper, we attempt to control the breakup characteristics of a circular liquid jet issuing from a 100 micron orifice. The breakup of the stream is investigated for different jet velocities and a composite disturbance is applied as the voltage signal to modify the breakup. The voltage signal applied to the piezoelectric terminals generates a vibration appearing in the form of an axial perturbation at the orifice exit which manifests as a radial disturbance on the surface of the jet. The perturbation grows along the jet and results in disintegration of the stream. At some frequencies and velocities, uniform breakup occurs and the droplets pinch off the liquid column at a constant rate. However, even in a uniform breakup, very small droplets (i.e. satellites) can form between the main droplets, which are not desired in many applications involving atomization and spray. The breakup is investigated experimentally over a frequency range of 4-18 kHz. In most previous experiments, a pure sinusoidal wave is applied to the piezoelectric and thereby the jet. In this work, we compare the breakup results from a single frequency disturbance with the results from a composite perturbation. The knowledge coming from this comparison is used to demonstrate how the breakup characteristics are controllable by adding extra modes to the driving disturbance. The experimental results corresponding to the jet subjected to a combination of disturbances also shows that the amplitude ratio of perturbation modes influences both the satellites merging and the breakup length.

Introduction

A capillary jet is a narrow stream emanating from an orifice. A disturbance is applied through an actuator onto the surface of the jet which generates waves on the surface of the moving fluid. As the liquid moves, the amplitude of the wave grows until it breaks the stream into droplets. The jet stream is under the influence of surface tension. The capillary instability is collapse of a liquid cylinder due to the capillary force, which is developed by the surface tension. In the 19th century, Plateau and Savart were the first who studied droplet formation in a capillary jet stream. In 1878, Rayleigh described the temporal instability of an infinitely long liquid column [1]. Rayleigh's work was modified in 1931 by Weber, who considered ambient fluid and liquid viscosity effects in his analysis and derived an equation for the growth rate of the instability as a function of the wavenumber [2]. However, comparing the experimental results with the findings from classical linear theories shows that the linear theory, while successful for some phenomena, cannot explain many behaviors of the jet stream to a high degree of accuracy. For example, the process of formation of satellites between main droplets is not predictable through linearization.

Experimental studies have been performed on satellite generation and control. Vassallo and Ashgriz studied the effect of amplitude and frequency on satellite formation [3]. However, using non-sinusoidal disturbances and additional harmonics as perturbation components has rarely been taken into account. Chaudhary investigated mechanisms of satellite merging and also considered the effect of second and third harmonic inputs in addition to the fundamental input on satellite control [4]. Huynh et al. also studied the nonlinear interaction between the components of a disturbance composed of different harmonics in order to control the droplets sizes and shapes [5]. Orme and Muntz also introduced amplitude-modulated disturbances to merge droplets produced from the fast frequency to form larger droplets [6, 7]. They break the stream into droplets with a frequency associated with a fast frequency applied to the actuator. Then, by applying an amplitude-modulated signal the droplets coalesce into a more stable configuration with the droplets frequency equal to the slow frequency of the modulating disturbance.

In these prior works, the disturbance applied to the jet surface is not measured and is assumed to be proportional to the input to the actuator. We experimentally measure the actual perturbation governing the jet by using a laser Doppler vibrometer.

*Corresponding author

In this paper, therefore, we investigate the breakup of the jet with the realistic disturbances that actually perturb the jet experimentally. We study the breakup of a jet of water at different velocities subjected to a combination of disturbances with different frequencies and amplitude ratios in order to control the breakup length and to eliminate satellites.

Disturbance Growth

According to the classical theories, for a disturbance to be unstable, the wavelength has to be greater than the circumference of the jet. In this case, the amplitude of the disturbance grows exponentially along the jet and results in separation of a droplet whenever it becomes as large as the radius of the jet. Following an unstable disturbance in time, the amplitude of the disturbance can be represented as $\varepsilon_0 e^{\beta t}$, where ε_0 is the initial disturbance amplitude and β is the growth rate of the disturbance along the jet. Weber derived an equation for the growth rate as a function of the wavenumber:

$$\beta^2 + \frac{3\mu k^2}{\rho_s r_s^{\,2}}\beta - \frac{\sigma k^2}{2\rho_s r_s^{\,3}}(1-k^2) - \frac{V_s^{\,2}\rho_a k^2 K_0(k)}{2\rho_s r_s^{\,2} K_1(k)} = 0, \tag{1}$$

where μ is the stream dynamic viscosity, ρ_s is the density of the liquid stream, k is the non-dimensional wave-number of the disturbance, r_s is the unperturbed jet radius, σ is the surface tension of the stream fluid, V_s is the stream speed, ρ_a is the ambient fluid density, and K_n is the n^{th} order Bessel function of the second kind.

As the equation shows, the growth rate is a function of the jet velocity. In Fig. 1, the growth rate versus the input or driving frequency is plotted based on the Weber's equation for the four different velocities we use.

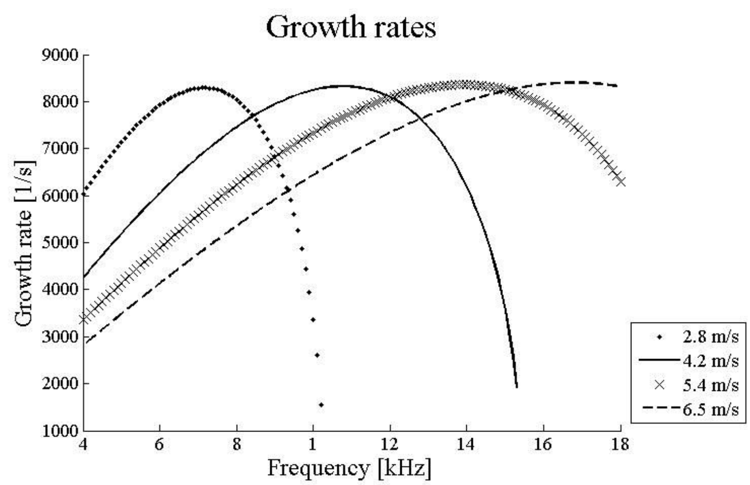

Figure 1. The growth rate as a function of the driving frequency for four jet velocities of 2.8 m/s, 4.2 m/s, 5.4 m/s, and 6.5 m/s.

Increasing the velocity shifts the growth rate plot to the right. Also, the graph is broadening as the velocity is increased and the peak of the plot is going up very slightly. Looking at the plots, we can predict that the breakup is more sensitive to driving frequency variation at lower velocities.

The growth rates are plotted for the unstable frequency range at each velocity and the frequency range is limited to 4-18 kHz according to the experimental setup constraints. Studying the growth rate plots at different jet speeds aids in understanding the growth rate effects on the breakup. For instance, the peak of the growth rate usually refers to a quick breakup due to a fast growing disturbance. A strong initial disturbance at a frequency associated with a high growth rate can eliminate the effect of interfering perturbations [8]. Experiments show that a strong disturbance leads to a uniform breakup for the jets moving at moderate velocities. In this work, uniform breakup is the representation of a breakup where the droplets are pinching off the stream at a constant frequency, i.e. the droplet frequency.

We have applied a pure sinusoidal perturbation at 4-18 kHz to a jet at the same condition but different velocities and captured the images of the drops for the unstable frequency range (Fig. 2). The velocities are the same as in Fig. 1. Fig. 2 (a) has the lowest speed (2.8 m/s) and Fig. 2 (d) corresponds to the highest speed (6.5 m/s).

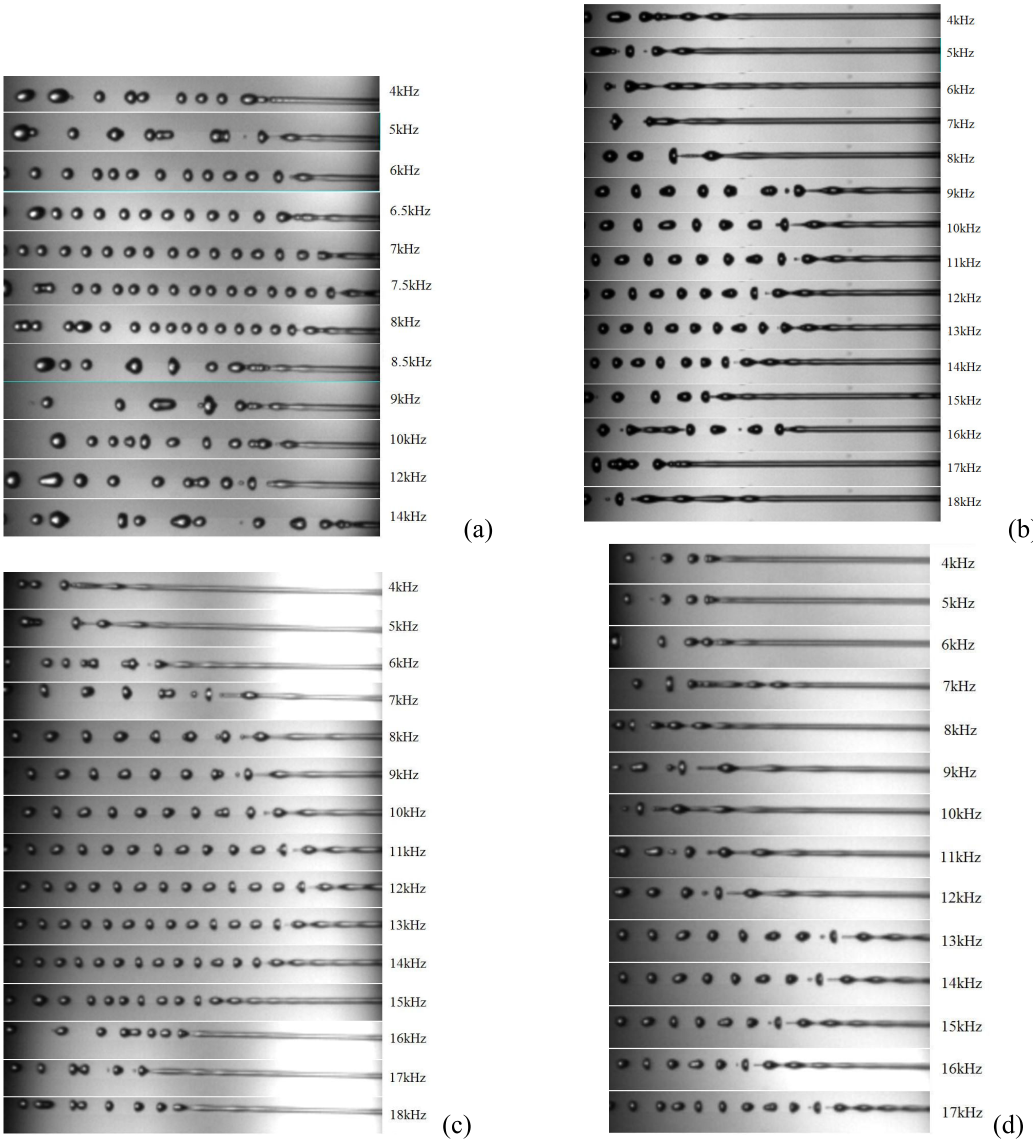

Figure 2. Breakup of a jet at different velocities: from 2.8 m/s in Fig. 2 (a) to 6.5 m/s in Fig. 2 (d).

As we can see from the figure, by increasing the velocity and broadening the growth rate graph, the frequency range associated with high growth rates becomes wider. Furthermore, merging of main droplets occurs further downstream due to low velocity of the jet as it can be seen at 6.5 kHz, 7.5 kHz, and 8 kHz in Fig. 2 (a) which corresponds to the lowest jet velocity, i.e. 2.8 m/s. This merging is more probable in lower velocities and also at lower frequencies since the distance between droplets is smaller.

Looking at the figure, we can see small droplets, i.e. satellites appearing between main droplets at the frequency of the growth rate peak. Around the growth rate peak, the breakup is uniform and the droplet frequency is still the same as the driving frequency (e.g. Fig. 2 (d), 11 kHz). It should be noted that the droplets frequency is the pinch-off frequency of the main droplets.

Off the peak of the growth rate, when the disturbance does not grow fast enough, not only is the breakup non-uniform and the droplet pinch-off at inconsistent frequency, but also the droplets are non-uniform in size. As we get close to the peak of the growth rate, it becomes more practical to eliminate the satellites and to control the breakup by manipulating the input. Later on we show that in this regime, the breakup is controllable to some extent by sending a composite disturbance (i.e. a disturbance consisting of multiple frequency perturbations with different amplitudes) to the jet.

Experimental Setup

There are various methods for generating disturbances on the jet. In this work, in order to generate droplets of about 100 microns, a vibration based method is used. Furthermore, the nozzle geometry is one of the parameters that characterize the breakup. Different velocity profiles caused by different nozzle geometries, result in changes in the breakup characteristics of the liquid jet. Dabiri and Sirignano have performed numerical studies on the effect of the nozzle geometry on a circular jet [9].

The droplet generator used in our work is a vibrating orifice of the Bergland-Liu type using a 100 micron orifice [10]. There is an orifice coupled with a piezoelectric, with the orifice located inside a donut shaped piezoelectric crystal. The voltage signal to the piezoelectric causes it to vibrate. The schematic of this design is shown in Fig. 3. As the stream passes through the orifice, the piezoelectric vibrates and axially applies the disturbance onto the stream by moving the circular jet along its axis.

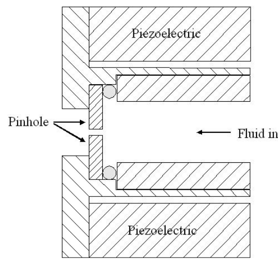

Figure 3. Schematic of the orifice inside the donut shaped piezoelectric crystal, mounted on the droplet generator.

A water-filled reservoir pressurized by nitrogen gas, supplies water to the droplet generator at a constant flow rate. In order to measure the vibration of the piezoelectric which is the actual perturbation on the jet, a MetroLaser Vibromet500 laser Doppler vibrometer (LDV) is used. We define the amplitude and the frequency of the voltage signal by an arbitrary function generator and generate a voltage signal consisting of multiple periodic sine waves at different frequencies with different amplitudes. The data points of the voltage signal are generated in Mathematica as a text file, in the format required for the function generator software and are sent to the droplet generator. Then, the actual vibration of the piezoelectric induced with the voltage signal is measured to find the real perturbation on the surface of the jet. As we discussed earlier, the actual disturbance on the jet is never measured and the perturbation applied to the jet is usually assumed to be proportional to the voltage signal.

Composite Disturbance's Effects on Breakup

Generally a disturbance with a high growth rate suppresses the effects of the disturbances with low growth rates. If the driving frequency is projected into a low growth rate in the growth rate graph, by adding a strong disturbance (with very high growth rate) we eliminate the effect of the driving perturbation and the breakup will be governed by the additional mode. This situation is shown in Fig. 4 where the driving frequency is 5 kHz and breakup is not uniform (Fig. 4 (a)). When an additional fast mode at 10 kHz (Fig. 4 (d)) or at 12 kHz (Fig. 4 (e)) is added to the disturbance components, the effect of the driving mode is suppressed and the breakup frequency is equal to the frequency of the additional mode at 10 kHz or 12 kHz. Comparing Fig. 4(d) with 4(b) and also 4(e) with 4(b) confirms that the driving perturbation at 5 kHz has only a slight influence on the breakup length.

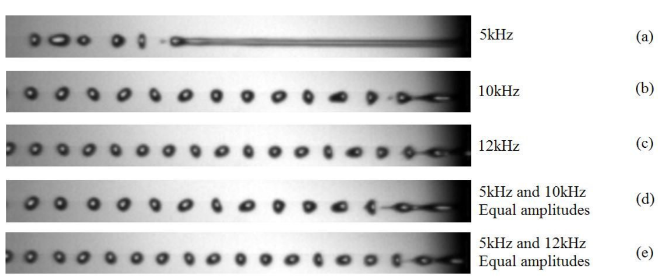

Figure 4. The breakup due to a pure wave at 5 kHz (a), 10 kHz (b) and 12 kHz (c) compared to a combination of 5 kHz and 10 kHz (d) and a combination of 5 kHz and 12 kHz (e).

However, if the driving mode is in the region where the growth rate is close to the peak and is not very low, adding extra modes with higher growth rates are useful in merging the satellites and producing uniform droplets. In this case, a proper amplitude ratio has to be chosen.

Besides being effective only in near the growth rate peak, there exists a narrow velocity range where we can take advantage of this merging behavior. At lower velocities where the growth rate plot is narrow, there is a fast transition from random breakup to a uniform breakup that occurs at the growth rate peak. Thus, sweeping the frequency toward the peak, at a frequency close to the peak where a uniform breakup occurs and satellites form, adding a strong mode usually changes the breakup frequency (as occurred in Fig. 4) since there exists a larger difference between the growth rates. On the other hand, at higher velocities, at a frequency where the breakup is uniform and satellites form, the growth rate of a strong mode does not seem to be much higher than the driving mode's growth rate and it cannot force the satellites to merge at the pinch-off.

Therefore, there exists an optimal velocity range where adding an extra mode will merge the satellites and result in uniform droplet formation. For example, if we project the growth rate of an 8 kHz driving frequency for a jet moving at 5.4 m/s Fig. 1 and look at the breakup over the unstable frequency range in Fig. 2 (c), we find this driving frequency a proper case for study. The breakup is uniform, while satellites form between main droplets (Fig. 5 (a)). If we add another mode at 16 kHz with an amplitude ratio of 0.75, it creates smaller droplets at higher frequency that merge after some time (Fig. 5 (b)). By increasing the amplitude ratio, as Fig. 5 (b), 5 (c), and 5 (d) show, we shift droplets merging toward the pinch-off position. Eventually, the optimum amplitude ratio is found experimentally to be 3 as it is shown in Fig. 5 (e) the droplets merge right after the pinch-off, which allows uniform droplets separating from the stream at the driving frequency, which was 8 kHz. It can be seen from the figure that the satellites produced by the pure 8 kHz are eliminated when the additional component is added.

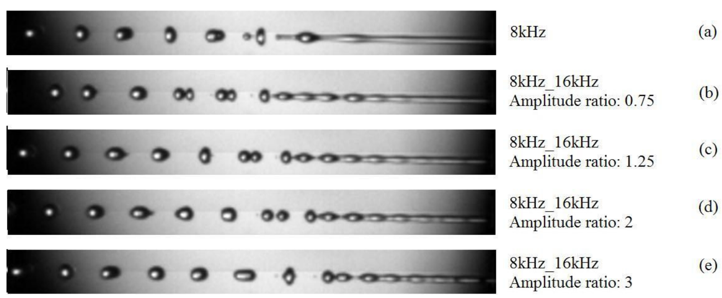

Figure 5. Increasing the amplitude ratio of an additional mode at 16 kHz and the driving frequency at 8 kHz, shifts droplets merging toward pinch-off. Satellites are also eliminated when the disturbance is composite.

The same process occurs at 9 kHz for the same jet moving at the same velocity (Fig. 1, shows the growth rate plot of a jet moving at 5.4 m/s). A pure sinusoidal wave at 9 kHz would produce satellites even though the droplets are pinching off at a constant rate of 9 kHz. Using a combination of 9 kHz and 18 kHz (considering the growth rates of two frequencies from Fig. 1 leads to satellite elimination by forcing them to merge. As Fig. 6 (b) to 6 (d) shows, by increasing the amplitude ratio we are indeed decreasing the size of the satellites formed between main droplets until they completely disappear in Fig. 6 (d) for a 0.2 amplitude ratio. By increasing the amplitude ratio higher than 0.2, the satellites appear again (at very small sizes) but droplet merging is shifted toward the pinch-off position.

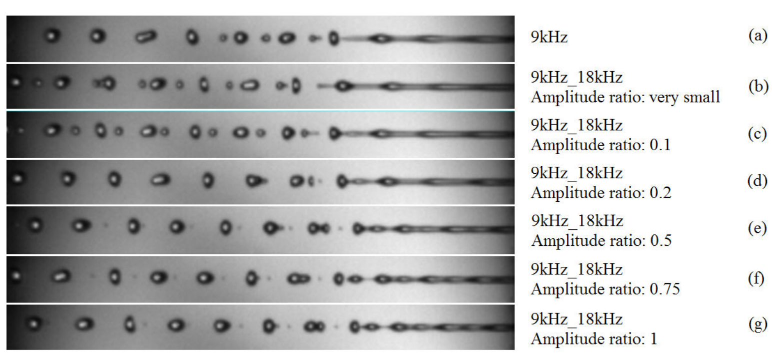

Figure 6. . Increasing the amplitude ratio of an additional mode at 18 kHz and the driving frequency at 9 kHz shifts droplets merging toward pinch-off.

Furthermore, our results show that amplitude-modulated disturbances can be used in the breakup length control. Breakup of a composite disturbance consisting of the driving mode at 12 kHz and an additional mode with an amplitude ratio of 1 is studied (see Fig. 1 for the growth rate and Fig. 2 for breakup of a jet moving at 6.5 m/s). At different frequencies for the additional mode, the breakup length is changed but the droplets frequency and the general form of the breakup are not changed. As Fig. 7 shows, increasing the frequency of the additional component from 6 kHz to 18 kHz (Fig. 7 (a) to (c)), the length of the breakup decreases. Hence, choosing a suitable driving mode with a high growth rate and a proper amplitude ratio, we can control the breakup length at a constant droplet frequency.

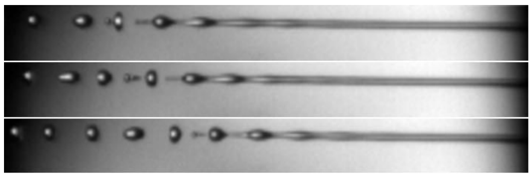

Figure 7. Controlling the breakup length using a composite disturbance for a 12 kHz driving frequency and additional modes at 6 kHz (a), 8 kHz (b), and 12 kHz (c).

Our results have some similarities to the amplitude-modulated work of Orme and Muntz [6, 7] who also perform droplet merging, however, their approach and goal are different. They apply a fast carrier frequency chosen in the unstable range and break the jet into small uniform droplets labeled as carrier droplets. When an amplitude-modulated disturbance is applied, the velocity variation due to an impulse exerted on the droplets in the direction of the breakpoint causes merging of the carrier droplets into larger droplets. Here, we control merging satellites to droplets by applying an amplitude-modulated disturbance. Hence, the sizes of the main droplets before and after applying the disturbance is not changing. Also, the typical merging time in the Orme and Muntz experiment is much greater than the breakup time. Merging occurs about 60 cm after traveling 30,000 droplets (as in [6] and [7]). We attempt to merge satellites and produce uniform droplets within a few droplet diameters from pinch-off.

Summary and Conclusions

In this paper, we study the breakup of a 100 micron circular jet of water at different velocities. The effects of a composite perturbation on breakup are studied from different aspects. Examples of changing the breakup length, merging the satellites to the main droplets and producing uniform droplets are represented to show how adding extra modes to the driving perturbation can be helpful in breakup control. The results come from the differences between growth rates of each mode that can be derived qualitatively from the Weber equation for the growth rate versus the frequency. Some of the perturbations are very strong and can dominate the effects of other perturbations, and some are weak and have no significant influence on the breakup. Combining different disturbances with different strengths can be useful in controlling the characteristics of a breakup. The growth rate plots at different velocities illustrate the feasibility of breakup control by composite disturbances. In the future, we will extend our approach to generate uniform droplets and spacing for wider frequency ranges off the growth rate peak.

Acknowledgement

This work is supported by the National Science Foundation under grant CMMI-0727609 and additional student support was provided by the University of California, Irvine.

References

1. Lord Rayleigh, *Proceedings of the London Mathematical Society*, 10:4-13 (1878).
2. Weber, C., *Ztschr. f. angew. Math und Mech*, 11:136-154, (1931).
3. Vassallo, P., and Ashgriz, N., *Proceedings Mathematical and Physical Sciences*, 433:269-286, (1991).
4. Chaudhary, K., C., PhD Dissertation, (1977).
5. Huynh, H., Ashgriz, N., and Mashayek, F., *Journal of Fluid Mechanics*, 320:185-210, (1996).
6. Orme, M., and Muntz, E., P., *Physics of Fluids A*, 2(7):1124-1140, (1990).
7. Orme, M., *Physics of Fluids A*, 3(12) :2936-2947, (1991).
8. Rohani, M., Dunn-Rankin, D., Jabbari, F., *21st Annual ILASS-Americas Conference on Liquid Atomization and Spray Systems,* Orlando, FL, May 2008.
9. Dabiri, S., and Sirignano, W. A., *20th Annual ILASS-Americas Conference on Liquid Atomization and Spray Systems,* Chicago, IL, May 2007.
10. Bergland R. N. and Liu B. Y. H., *Environ. Sci. Tech.*, 7:147-153, (1973).